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25th BATCH

COMPUTER AND COMMUNICATION ENGINEERING

International Islamic University Chittagong

COURSE CODE: CCE-3511

COURSE TITLE: Electro Magnetic Field

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CCE-3511

(Electro Magnetic Field & wave)

Magnostatic Field :-

Steady Magnetic Fields are Fields produce by steady current. These are the Fields which are constant with time.

Objectives :-

- App of MS Field
- Faraday's induction law, Biot-Savart law and Force law for current element
- Energy stored in Magnetic Field

EM waves :-

Objectives :-

- Application of EM wave
- wave equations & Solutions
- wave propagation characteristics
- wave in conductors, & dielectrics (Materials that does not allow element)

Electrostatic Fields:

ES Fields are also called static electric Fields or steady electric Field. These are not variant with time. They are produced by static charges on charge distribution.

Objective: Electrostatic:

- → Applications of electrostatic
- → Charge Distribution
- → Coulombs law + Application + limit
- → Gauss's law + "
- → poissions & Laplace equations
- → potential Function
- → Energy stored in ES Field.

~~Ohm's Law~~ (Coulomb's Law)

$$\vec{F} \propto \frac{Q_1 \cdot Q_2}{(r)^2} \vec{a_r}$$

=> Coulomb's Law states that there exists a force between charged bodies & it is

-> proportional to the product of the two charges

-> Inversely proportional to the square of the distance between the charges

The force also depends on the medium in which the charges are located. The force is a vector quantity & it is attractive if the charges are unlike & repulsive if the charges are alike.

Mathematically,

$$F \propto \frac{Q_1 \cdot Q_2}{r^2} \times a_r$$

$$F = K \cdot \frac{Q_1 \cdot Q_2}{r^2} \times a_r$$

where K = a Constant of proportionality $= 1/4\pi\epsilon$

Unit = N (newton)

ε = permitivity of the medium in which the charges are located (F/m)

$= \varepsilon_0 \varepsilon_r$

ε_0 = Free space (permitivity)

$= \frac{1}{36\pi} \times 10^{-9}$

$= 8.854 \times 10^{-12}$ F/m

ε_r = relative permitivity of the medium with respect to free space (has no unit)

K in free space

$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}$ m/F

a_r = Unit vector along the line joining the two charges.

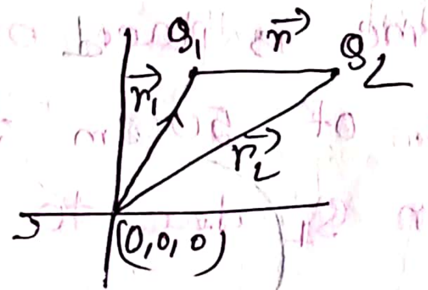
Force ~~of~~ on q_2 due to q_1 in Free space is written in the form of

$$F_{21} = \frac{q_1 \cdot q_2}{4\pi\epsilon_0 (r_{21})^2} \times \hat{a}_{21}$$

$$\vec{F_{12}} = -\vec{F_{21}}$$

Force ~~of~~ on q_1 due to q_2

$$F_{12} = \frac{q_1 \cdot q_2}{4\pi\epsilon_0 (r_{12})^2} \times \hat{a}_{12}$$



$$\underset{(x_1, y_1, z_1)}{q_1} \xrightarrow{\vec{r}} \underset{(x_2, y_2, z_2)}{q_2}$$

$$\vec{r_1} = x_1 \hat{a}_x + y_1 \hat{a}_y + z_1 \hat{a}_z$$

$$\vec{r_2} = x_2 \hat{a}_x + y_2 \hat{a}_y + z_2 \hat{a}_z$$

$$\vec{r_{21}} = (x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z$$

$$|\vec{r}_{21}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

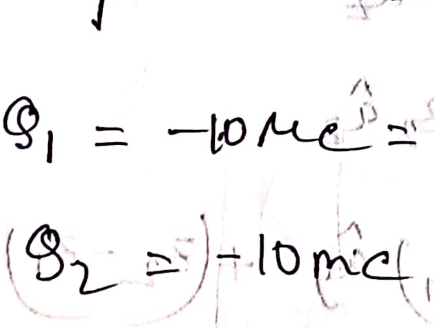
$$\vec{r}_{12} = (x_1-x_2)\,\hat{a}_x + (y_1-y_2)\,\hat{a}_y + (z_1-z_2)\,\hat{a}_z$$

$$|\vec{r}_{12}| = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$$

Math -01

5 A charge of $Q_1 = -10\ \mu c$ is placed at the origin of a rectengular coordinate system & a second charge, $Q_2 = -10\ mc$ is placed on the x-axis at a distance of 50 cm from the origin. Find the force on Q_1 due to Q_2 if they are in free space.

cm এ থাকলে meter এ নিতে হবে
0.5 m হবে
Answer $-360 a_x$ N



$Q_1 = -10\ \mu c = -10 \times 10^{-6}$ C

$Q_2 = -10\ mc = -10 \times 10^{-3}$ C

$\vec{F_{12}} = ?$

we know

$$F_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r_{12}^2} \times \hat{a}_{12}$$

$$r_1 = 0.\hat{a}_x + 0.\hat{a}_y + 0.\hat{a}_z$$

$$r_2 = 50.\hat{a}_x + 0.\hat{a}_y + 0.\hat{a}_z$$

$$\vec{r_{12}} = (0-50)\hat{a}_x + (0-0)\hat{a}_y + (0-0)\hat{a}_z$$

$$= -50\,\hat{a}_x$$

$$|\vec{r_{12}}| = \sqrt{(-50)^2} = 50$$

$$\therefore r_{12} = 50$$

$$\hat{a}_{12} = \frac{\vec{r_{12}}}{|\vec{r_{12}}|} = \frac{-50}{50}\hat{a}_x = -\hat{a}_x$$

$$\therefore \vec{F_{12}} = \frac{-10\times10^{-6}\times10\times10^{-3}\times9\times10^{9}}{(50)^2}\times(-\hat{a}_x)$$

$$= -0.036\,\hat{a}_x \text{ Newton}$$

Field due to surface charge Density:-

Field due to surface density,

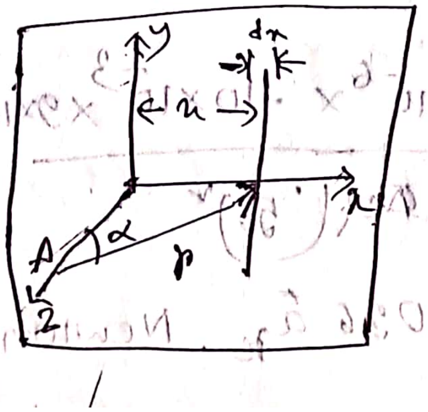
$$\vec{E} = \frac{\rho_s}{2\varepsilon_0} \hat{a}_n$$

Consider an infinity charged sheet lying in x-y plane. Assume that the sheet is uniformly charged.

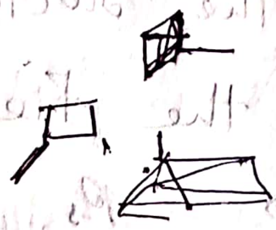
Consider a strip of a differential width of dx.

The sheet extends from $-\infty$ to ∞ in both x & y directions. It is obvious that the field does not vary with x or y due to symmetry. Now there is only z component.

By definition, $\rho_s = \frac{dQ}{dS}$



⇒ A differential strip in an infinite surface charged sheet

$dQ = \rho_s \, dS$ 

$= \rho_s \, dx \, dy$

That is, $\dfrac{dQ}{dy} = \rho_s \cdot dx$

For a differential strip of width dx we have $\rho_L = \rho_s \, dx$

The field at a point, A on z axis is given by,

$$dE = \frac{\rho_s \, dx}{2\pi \varepsilon_0 r} \hat{a}_r$$

$$\therefore dE_z = \frac{\rho_s \, dx}{2\pi \varepsilon_0 r} \cdot \cos\alpha$$

Here, $r = \sqrt{z^2 + x^2}$

$\cos\alpha = z/r$

$$\therefore E_z = \frac{\rho_s}{2\pi\varepsilon_0} \int_{-\infty}^{\infty} \frac{z^2}{(z^2+x^2)} dx$$

$$= \frac{\rho_s}{2\pi\varepsilon_0} \left[\tan^{-1} x/z \right]_{-\infty}^{\infty}$$

that is, $E_z = \dfrac{\rho_s}{2\varepsilon_0}$; $E_z = \dfrac{\rho_s}{2\varepsilon_0} a_z$

* If the surface charged sheet lies in y-z plane, the field at a point on x-axis is

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\,\hat{a}_x$$

* Similarly, if it is ~~on~~ in x-z plane, the field at a point on y-axis is,

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\,\hat{a}_y$$

In general, the field at a point on the axis normal to the plane of the sheet is given by,

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\cdot\hat{a}_n$$

Uniform charge Density

Math :

An infinite sheet in x-y plane extending from $-\alpha$ to α in both direction has a uniform charge density of $10\ nC/m^2$. Find the electric field at $z = 1.0$ cm

=> We know,

$$\vec{E} = \frac{\rho_s}{2\epsilon_0} \times \hat{a}_z$$

$\rho_s = 10\ nC/m^2$

$= 10 \times 10^{-9}\ Cm^{-2}$

$\epsilon_0 = 8.854 \times 10^{-12}$

$$= \frac{10 \times 10^{-9}}{2 \times 8.854 \times 10^{-12}}\ \hat{a}_z$$

$$= 564.717 a_z\ V/m$$

$\boxed{\text{Field due to volume charge Density}}$

Determination of field due to volume charge density simply involves the estimation of total charge, Q from ρ_v,

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \cdot \hat{a}_r \quad \Big| \quad E = \int_v \frac{\rho_v}{4\pi\epsilon_0 r^2}\, dv\ \hat{a}_r \left(\frac{N}{C}\right)$$

Potential

potential at a point due to a fixed charge is defined as the work done in bringing one coulomb of charge from infinity to the point against the force created by the fixed charge, that is, the potential is work done per unit charge.

$$V \cong \frac{\text{work done to bring a charge } q \text{ from } \infty \text{ to the point towards } q_f}{q}$$

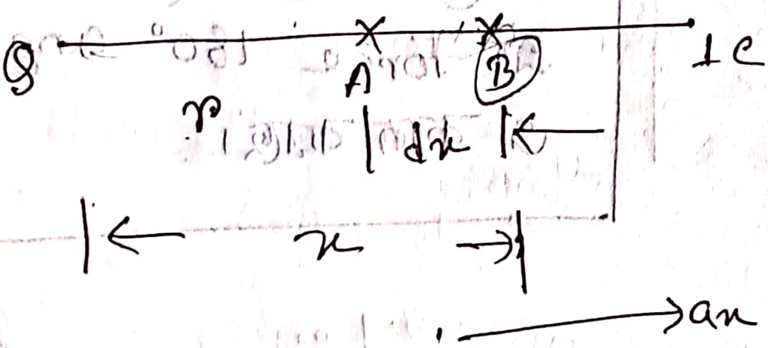
Simply, $V \cong \dfrac{\text{work Done}}{q}$, JC^{-1} or, volt

potential at a point

The potential at a point due to a point charge is given by,

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

potential at a point:-



Note:-

* consider a fixed charge, Q and 1C of charge at an infinite distance. There exists a force on 1C, by Q. If 1C of the charge is moved against the force of repulsion, some work has to be done.

* If 1C is at point B which is at a distance of x from Q, then force on 1C due to Q is,

$$F = \frac{Q}{4\pi\epsilon_0 \cdot x^2} \cdot a_x$$

Proof:-

$$F = \frac{q}{4\pi\epsilon_0 x^2}$$

Here,

$$W = FS$$

$$\therefore dW = F\,dx \cos 180^\circ$$

$$= -F\,dx$$

Note:-
q charge এর দিকে Force দিচ্ছে এবং 1C charge, Force দিচ্ছে তার বিপরীত দিকে। যার ফলে তাদের এই Force 180° angle এ কাজ করছে।

So,

$$\int dW = \int_{\infty}^{r} -F\,dx$$

$$W = \int_{\infty}^{r} -\left(\frac{q}{4\pi\epsilon_0 x^2}\right) \cdot dx$$

$\frac{q}{4\pi\epsilon_0}$ → constant

$\frac{1}{x^2} = x^{-2}$

$\frac{x^{n+1}}{n+1}$ $\frac{x^{-3}}{-3+1}$

$$= -\frac{q}{4\pi\epsilon_0} \int_{\infty}^{r} \frac{1}{x^2}\,dx$$

$$= -\frac{q}{4\pi\epsilon_0} \int_{\infty}^{r} x^{-2}\,dx$$

$$= -\frac{q}{4\pi\epsilon_0}\left[-\frac{1}{x}\right]_{\alpha}^{r} \quad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1} \quad \int x^{-2} dx = \frac{x^{-2+1}}{-2+1} = -\frac{1}{x}\right]$$

$$= -\frac{q}{4\pi\epsilon_0}\left[-\frac{1}{r} - \left(-\frac{1}{\alpha}\right)\right]$$

$$= -\frac{q}{4\pi\epsilon_0}\left[\left(-\frac{1}{r}\right)\right]$$

$$\therefore W = \frac{q}{4\pi\epsilon_0 r}$$

Hence, $V = \frac{q}{4\pi\epsilon_0 r}$

Math

A charge of 10 PC is at rest in free space. Find the potential at a point A 10 cm away from the charge.

⇒

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

$$V = \frac{1}{4\pi\epsilon_0} \times Q/r$$

we know,

$$\frac{1}{4\pi\epsilon_0} = 9\times10^9$$

$$\therefore V = \frac{10\times10^{-12}}{0.1} \times 9\times10^9$$

$$= 0.9V$$

Here,

Q — C(10pc)

$r = 10cm$ $10\times10^{-12}C$

$= 0.1m$

~~Q = 8.854 x.~~

$Q = 10pc$

$= 10\times10^{-12}C$

$r = 10cm$

$= 0.1m$

Potential Difference

(বিভৱ পাৰ্থক্য)

* The potential difference between two points A & B is defined as the work done by an applied force in moving a unit positive charge from A to B in electric field.

The work done $W = -q \int_A^B E \cdot dl$

$$V_{AB} = \frac{W}{q} = -\int_A^B E \cdot dl$$

q charge moved from A to B

* potential difference between A & B also described ~~the~~ as the difference between the potentials at A & B.

$$V_A = -\int_{\infty}^{A} E \cdot dL$$

$$V_B = -\int_{\infty}^{B} E \cdot dL$$

$\therefore V_{AB} = V_A - V_B$

potential Gradient

Potential Gradient = ∇V

∇ = vector differential operator & V is scalar potential.

$\boxed{\text{Formula}}$

⇒ The potential gradient in different coordinate system is given by,

(i) $$(\nabla V)_{\text{Cartesian}} = \frac{\delta V}{\delta x} a_x + \frac{\delta V}{\delta y} \cdot a_y + \frac{\delta V}{\delta z} \cdot a_z$$

(ii) $$(\nabla V)_{\text{cylindrical}} = \frac{\delta V}{\delta \rho} \hat{a}_\rho + \frac{1}{\rho} \frac{\delta V}{\delta \varphi} \cdot a_\varphi + \frac{\delta V}{\delta z} \hat{a}_z$$

(iii) $$(\nabla V)_{\text{spherical}} = \frac{\delta V}{\delta r} \hat{a}_r + \frac{1}{r} \frac{\delta V}{\delta \theta} \cdot \hat{a}_\theta + \frac{\delta V}{r \sin\theta \, \delta \varphi} \cdot a_\varphi$$

Math

problem:- If the potential function, V is given by $V = x^3y - xy^2 + 3z$, Find the potential gradient.

Solution:-

$$V = x^3y - xy^2 + 3z$$

we know,

$$\nabla V = \frac{\delta V}{\delta x}\hat{a}_x + \frac{\delta V}{\delta y}\hat{a}_y + \frac{\delta V}{\delta z}\hat{a}_z$$

$$\frac{\delta V}{\delta x} = \frac{\delta}{\delta x}(x^3y - xy^2 + 3z)$$

$$= 3x^2y - y^2$$

$$\frac{\delta V}{\delta y} = \frac{\delta}{\delta y}(x^3y - xy^2 + 3z)$$

$$= x^3 - 2xy$$

$$\frac{\delta V}{\delta z} = \frac{\delta}{\delta z}(x^3y - xy^2 + 3z)$$

$$= 3$$

$$\therefore \nabla V = (3x^2y - y^2)\hat{a}_x + (x^3 - 2xy)\hat{a}_y + 3\hat{a}_z \ Vm^{-1}$$

(Ans)

Electric Flux

Electric Flux is also known as Electric Displacement Flux.

→ It is defined as the displaced charge, that is Electric Flux, $\psi = Q$, coulomb

→ Electric Flux is defined as the surface integral of electric flux density, that is,

Electric Flux, $\psi = \int_s D \cdot ds$

Electric Flux Density

Definition:-

⇒ Electric Flux Density, D is defined as,

$$D = \frac{d\psi}{ds} \cdot a_n \;, \; C/m^2$$

where ψ is the electric flux according crossing the differential area, dS. The direction of ds is always outward (बाहर की ओर), normal to ds,

That is $d\mathbf{s} = dS\,a_n$

Definition-02

Electric ~~Field~~ Flux density, D

$D = \epsilon E$, $C\,m^{-2}$

ϵ_0 = permitivity (অনুমতি) of free space, F/m

E = Electric field strength, V/m

$\boxed{\text{Math}}$

* If an electric field in free space is given by, $E = a_x + 2a_y + 5a_z$ V/m. Find the electric ~~Field~~ Flux density.

$\Rightarrow$ Electric field, $E = a_x + 2a_y + 5a_z$

$\epsilon_0 = 8.854 \times 10^{-12}$ F/m

$D = \epsilon_0 \cdot E$

$= 8.854 \times 10^{-12} (a_x + 2a_y + 5a_z)$

$D = (8.854 a_x + 17.7 a_y + 44.27 a_z)$, pC/m²

Gauss's law & Applications

It states that the net flux passing through any closed surface is equal to the charge enclosed by that surface.

(কোনো বদ্ধ ক্ষেত্র হতে discharge হওয়া চার্জের সম্পূর্ণ পরিমান = ঐ বদ্ধ ক্ষেত্রের বিদ্যমান সমস্ত চার্জ)

$$\oint_S D \cdot ds = Q_{enc} \rightarrow \text{enclosed}$$

⇒ It is known as Gauss's law in integral form.

And it is also applied in Gaussian surfaces.

A spherical surface which encloses a charge Q at its centre.

The differential area ds, is on the surface of the sphere whose direction is a_n. Let r be the radius of the sphere.

The electric field E, at the spherical surface is given by,

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \cdot a_r$$

But, $D = \varepsilon_0 E = \frac{Q}{4\pi r^2} \cdot a_r$

Taking dot product with ds on both sides, we get,

$$D \cdot ds = \frac{Q}{4\pi r^2} \cdot a_r \cdot ds \cdot a_n$$

Here,

a_r & a_n have the same direction

$$\therefore D \cdot ds = \frac{Q}{4\pi r^2} \cdot ds$$

Taking surface integral on both sides, we get,

$$\oint D \cdot ds = \int_s \frac{Q}{4\pi r^2} \cdot ds = \frac{Q}{4\pi r^2} \cdot \oint_s ds$$

$$\pm \frac{Q}{4\pi r^2} \cdot S$$

But,

$S = 4\pi r^2$, area of spherical, so,

$$\oint D \cdot ds = Q$$

$\boxed{\text{Hence proved}}$

Poisson's & Laplace's Equations

Poissons And Laplace Equation

=> poisson হতে Laplace আসবে

Poisson's equation,

$$\nabla^2 V = -\frac{\rho_v}{\epsilon}$$

Laplace equation,

$$\nabla^2 V = 0$$

Proof:-

The point form of Gauss's law is,

$$\nabla \cdot D = \rho_v$$

But, $D = \varepsilon E$

$E = -\nabla V$

and, $\nabla \cdot D = \nabla \cdot \varepsilon E$

$= \nabla \cdot \varepsilon(-\nabla V) = \rho_v$

$\therefore \nabla^2 V = -\frac{\rho_v}{\varepsilon}$

Here, ∇^2 is a scalar operator $\left(\frac{1}{m^2}\right)$ and is called as Laplace's operator,

$$\nabla^2 = \frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} + \frac{\delta^2}{\delta z^2}$$

In the region where, $\rho_v = 0$, Poisson's equation becomes,

$$\nabla^2 \cdot V = 0$$

Laplace's equation in one dimension,

$$\nabla^2 V = \frac{\delta^2 V}{\delta x^2} = 0$$

Laplace's equation on two dimension,

$$\frac{\delta^2 V}{\delta x^2} + \frac{\delta^2 V}{\delta y^2} = 0 = \nabla^2 V$$

Laplace's equation on three dimension,

$$\nabla^2 V = \frac{\delta^2 V}{\delta x^2} + \frac{\delta^2 V}{\delta y^2} + \frac{\delta^2 V}{\delta z^2} = 0$$

Boundary Condition on E & D

① The tangential component of E is continuous across any boundary, that is,

$$E_{tan1} = E_{tan2}$$

or,

The tangential component of E in medium 1 is the same as that of E in medium 2 at any boundary.

② The normal component of D is continuous across any boundary except at the surface of the conductor, In general,

$$D_{n1} - D_{n2} = \rho_s$$

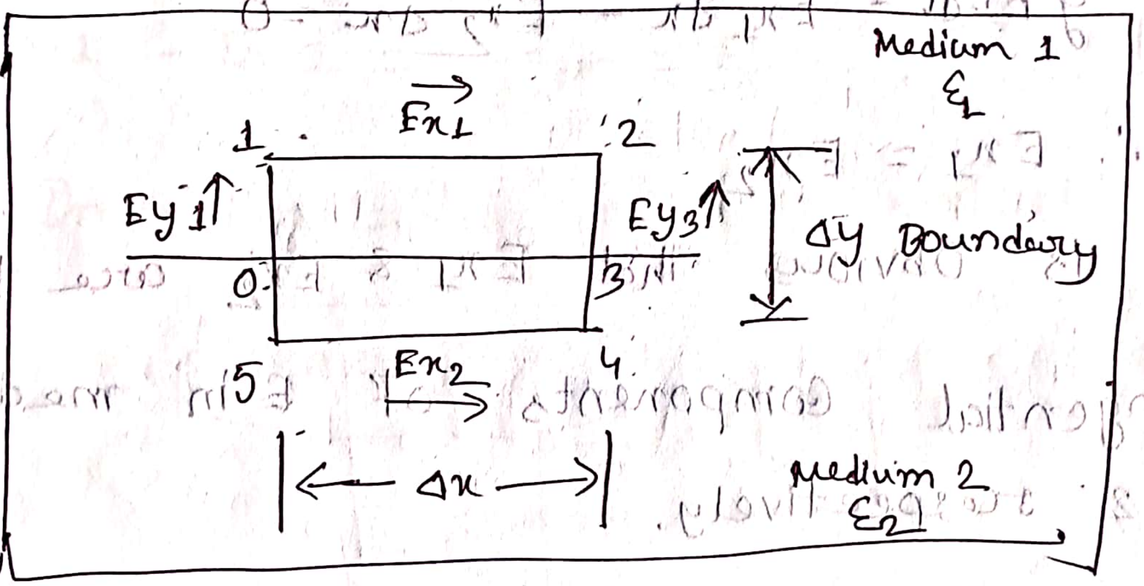
ρ_s = Surface charge density (Cm^{-2})

For any point other than the conductor.

~~bon~~ boundary, $D_{n1} = D_{n2}$

proof of boundary conditions:

Consider the rectangular loop on the boundary of two media,



Rectangular loop on boundary

It is well known that electric field is conservative & hence the line integral of $E \cdot dl$ is zero around a closed path,

$$\oint E \cdot dl = 0$$

From the above figure,

$$\oint_{l} E \cdot dl = \int_{01} + \int_{12} + \int_{23} + \int_{34} + \int_{45} + \int_{50}$$

$$= E_{y1}\frac{\Delta y}{2} + E_{x1}\Delta x - E_{y3}\frac{\Delta y}{2} - E_{y4}\frac{\Delta y}{2} - E_{x2}\Delta x + E_{y2}\frac{\Delta y}{2}$$

As $\Delta y \to 0$, we get,

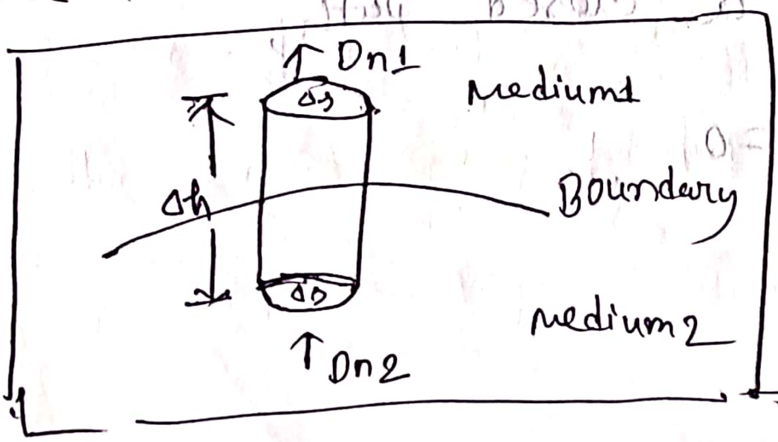
$$\oint E \cdot dl = E_{x1}\Delta x - E_{x2}\Delta x = 0$$

$$\therefore E_{x1} = E_{x2}$$

It is obvious that E_{x1} & E_{x2} are the tangential components of E in medium 1 & 2 respectively.

$$E_{tan1} = E_{tan2}$$

Now consider a cylinder across media 1 & media 2



cylindrical surface on boundary

According to Gauss's law,

$$\oint D \cdot ds = 0$$

Applying this to cylindrical surface on the boundary spreading over medium1 & medium2, we get, $\Delta h \to 0$

$$D_{n1} \Delta s - D_{n2} \Delta s = Q$$

$$D_{n1} - D_{n2} = \frac{Q}{\Delta s} = \rho_s$$

$$\therefore D_{n1} - D_{n2} = \rho_s$$

Hence proved

Electric Field / Electric Field Strength / Electric Field intensity

Electric Field due to a charge is defined as the Coulomb's Force per unit charge. It is a vector & has the unit of Newton per Coulomb or volt per metre, that is,

$$\text{Electric Field, } E = \frac{F}{Q} \text{ N/C}$$

Electric Field Strength Due to Point Charge:-

q_F = Fixed point charge, C

q_t = test point charge, C

r_F = Location of Fixed charge

r_t = Location of test Charge

Then the Force on q_t due to Fixed charge in Free space q_F is given by,

$$F_{tF} = \frac{q_t \cdot q_F}{4\pi\varepsilon_0 r_{tF}^2} a_{tF}, \ N/C$$

The electric Field, E at the location of q_t due to q_F is defined as the ratio of Force on q_t due to q_F and the test charge, q_t, that is,

$$E \cong \frac{F_{tF}}{q_t}$$

$$E = \frac{q_F}{4\pi\varepsilon_0 r_{tF}^2} \cdot a_{tF}, \ N/C$$

□

Electric Field due to line Charge Density:-

By definition, line charge density is given by,

$$\rho_L = \frac{dq}{dL}, \ C/m$$

$$dq = \rho_L \, dL$$

$$q = \int \rho_L \cdot dL$$

Here ρ is the total charge

But electric field due to ρ at a distance of r is given by,

$$E = \frac{\rho}{4\pi\epsilon_0 r^2} a_r$$

$$E = \int \frac{\rho_L \, dL}{4\pi \epsilon_0 r^2} \cdot a_r$$

KEEP
CALM
ITS TIME FOR THE
FINAL
EXAM

Steady Magnetic Fields

It is constant with time.

Steady currents produce steady magnetic fields. It is also called as magnostatic fields.

Magnetic field intensity, H

Magnetic flux density, B

Their relation, $B = \mu H$

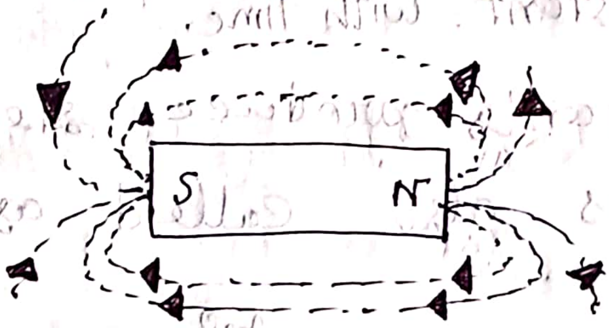
* Steady Magnetic Fields are governed by Biot-Savart-Law & Ampere's circuit law.

Fundamentals of Steady Magnetic Fields:-

Magnetic fields are also called static magnetic fields or magnetostatic fields. These are produced by a magnet or by a current element.

The two opposite ends of a magnet are called its poles.

Magnetic Lines Of Force / Flux:-



It is observed the ~~ion~~ iron fillings arrange themselves in a set of parallel lines going ~~goi~~ from one pole to another. These lines never cross or ~~o~~ unite. These are called the magnetic lines of force / Flux.

Magnetic Flux:-

Lines of force produced in the medium surrounding electric currents or magnets

Magnetic Flux density, (B), (Wb/m^2)

Line of Force, $\phi = (-)\int_S B \cdot ds$, weber

(–) হতে পারে
আবার (+) হতে পারে

$$\therefore B = \frac{d\phi}{ds} \cdot a_n$$

B is also defined as, $B = \mu H$

H = magnetic Field (A/m)

μ = permeability of the medium (H/m)

$= \mu_0 \cdot \mu_r$

μ_0 = permitibility of free space

$= 4\pi \times 10^{-7}$ Hm^{-1}

μ_r = Relative permeability of the medium.

Current Element:-

A current element is a conductor carrying current. It is represented by IL.

I = current

L = conductor.

[Math]

Magnatic field $H = 3a_x + 2a_y$ A/m exists at a point in free space, what is the magnetic flux density at the point?

$\Rightarrow$ Here, $H = 3a_x + 2a_y$

$\mu_0 = 4\pi \times 10^{-7}$

$\therefore B = \mu_0 H = (4\pi \times 10^{-7})(3a_x + 2a_y)$

$$= \left(3.76\, a_x + \cancel{7.54}\ 2.513\, a_y\right) \times 10^{-6} \quad wb/m^2$$

$$= 3.76\, a_x + 2.513\, a_y \quad \mu wb/m^2$$

(Ans)

Ampere's law for current element OR BIOT - SAVART LAW

Biot-Savart law is given by,

$$dH = \frac{IdL \times a_r}{4\pi r^2} \quad A/m$$

dH = Magnetic field at a point

IdL = differential current element (A-m)

a_r = Unit vector along the line joining the point P and the IdL

r = Distance of P from the current element (m).

Statement of Biot-Savart Law :-

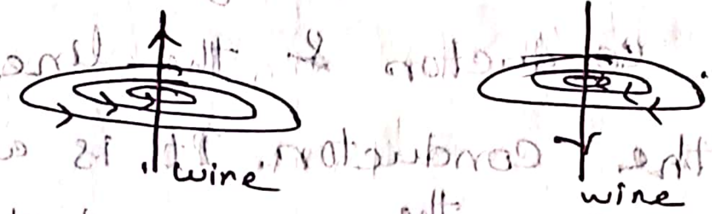
~~Curr~~ differential current produces a differential magnetic field, dH. The field magnitude at a point is proportional to the product of IdL, and sign of the angle between the conductor & the line of the point to the conductor. It is also inversely proportional to ~~the~~ the square distance from the element to the point.

If the current is upward, the direction of magnetic field is anti-clockwise and if the current is downward, the direction of magnetic field is clockwise.

To find it out easily we use right hand with the thumb.

If a current element is held in the right hand with the thumb pointing upwards indicating the direction of current, then the remaining fingers indicate the direction of the magnetic field.

If the current is upward, the direction of magnetic field is anticlockwise & if the current is downwards, the direction of magnetic field is clockwise.



Field due to infinitely long current Element:-

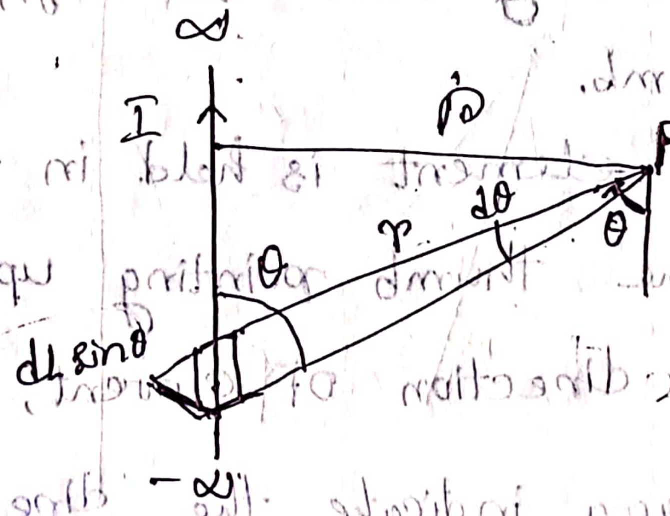
The field produced by an infinitely long current element at a point is given by,

$$H = \frac{I}{2\pi\rho} a_\varphi$$

I = Current in the element

ρ = Distance of the point from the element

proof:-



By Biot - Savart law we have,

dH due to IdL at P,

$$dH = \frac{IdL \times \hat{a}_R}{4\pi R^2}$$

$$= \frac{IdL \sin\theta}{4\pi r^2} \times \hat{a}_\phi$$

But $dL \sin\theta = r\,d\theta$

$$\therefore dH = \frac{I r\, d\theta}{4\pi r^2} \times \hat{a}_\phi$$

$$\therefore dH = \frac{I}{4\pi} \cdot \frac{\sin\theta}{\rho} \cdot d\theta\, \hat{a}_\phi \quad \left[\sin\theta = \frac{\rho}{r},\ \therefore \frac{1}{r} = \frac{\sin\theta}{\rho} \right]$$

So, H due to infinitely long current element is given by,

$$H = \frac{I}{4\pi\rho} \int_0^\pi \sin\theta \cdot d\theta \cdot \hat{a}_\phi$$

$$= \frac{I}{4\pi\rho} \left[-\cos\theta \right]_0^\pi$$

$$= \frac{I}{4\pi\rho} \left[-\cos(\pi) + \cos(0) \right] \hat{a}_\phi$$

$$= \frac{I}{4\pi\rho} [1+1]\, \hat{a}_\phi = \frac{I}{2\pi\rho}\, \hat{a}_\phi \ (A/m)$$

(Ans)

Field Due To Finite Current Element:-

$$H = \frac{I}{4\pi R}\left[\cos\alpha_2 - \cos\alpha_1\right] a_\phi$$

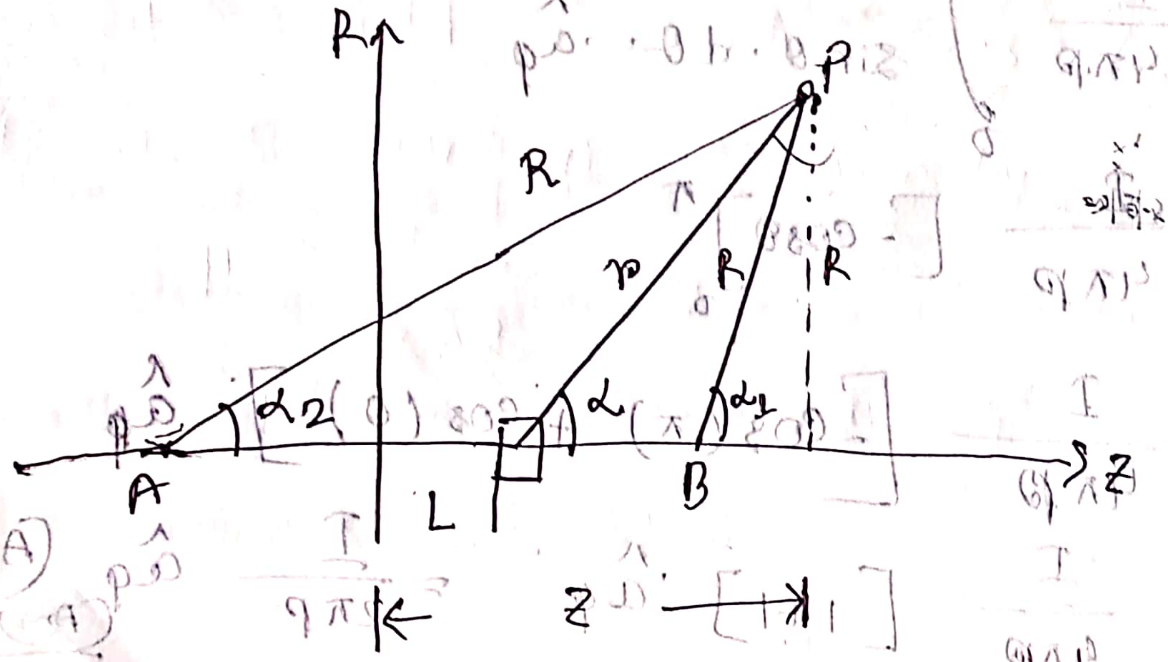
where,

I = Current in the element.

R = Distance of the point from the element axis,

α_1 = Angle made by the line joining the point and one of the element with the axis of the element.

α_2 = Angle made by the line joining the point and the other end of the element with the axis.



The differential magnetic field, dH, at the poin P due to IdL,

$$dH = \frac{IdL \sin\alpha}{4\pi r^2} a_\phi$$

$$H = \int_{0}^{a} \frac{Idl \sin\alpha}{4\pi r^2} a_\phi$$

we have, $r^2 = (Z-L)^2 + R^2$

$$Z - L = R\cot\alpha$$

Thus, we get $-dL = -R\operatorname{cosec}^2\alpha \, d\alpha$

$$dL = R\left[\frac{R^2 + (Z-L)^2}{R^2}\right] d\alpha$$

$$H = \frac{1}{4\pi R}\int_{\alpha_2}^{\alpha_1} \sin\alpha \, d\alpha \, a_\phi = \frac{1}{4\pi R}\left[-\cos\alpha\right]_{\alpha_2}^{\alpha_1} a_\phi$$

$$= \frac{1}{4\pi R}\left[\cos\alpha_2 - \cos\alpha_1\right] a_\phi \text{ , A/m}$$

[proved]

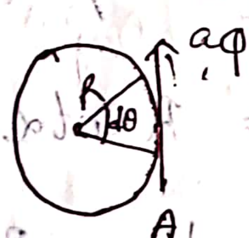
Ampere's work law or Ampere's circuit Law:-

Ampior's circuit Law:-
The line integral of the magnetic field H about any closed loop is equal to the current enclosed by the path.

Mathematically,

$$\oint H \cdot dL = I_{enc}$$

proof:-



$$dL = dL\, a_\phi$$

Consider a circular loop as in figure which enclosed a current element. Let the current be in upward direction. Then the field is anticlockwise. (a_ϕ)

H at the point A is given by

$$H = \frac{I_{enc}}{2\pi R} a_\phi$$

Taking dot product with dL on both sides, we get

$$H \cdot dL = \frac{I_{enc}}{2\pi R} a_\phi \cdot dL a_\phi$$

$$= \frac{I_{enc}}{2\pi R} dL \qquad [\hat{a} \cdot \hat{a} = 1]$$

But, $dL = dR d\phi$

$$\Rightarrow H \cdot dL = \frac{I_{enc}}{2\pi R} \cdot R \, d\phi$$

$$\Rightarrow H \cdot dL = \frac{I_{enc}}{2\pi} d\phi \quad (1)$$

$$\Rightarrow \oint H \cdot dL = \int_0^{2\pi} \frac{I_{enc}}{2\pi} d\phi$$

$$\Rightarrow \oint H \cdot dL = I_{enc}$$

This is called the ~~ino~~ integral form of Ampere's circuit law.

Differential Form of Ampere's Circuit Law:-

The law is given by,

$$\nabla \times H = J$$

J = ~~Cot~~ conduction current density (A/m^2)

Maxwell's 3rd equation in integral and differential form:-

According to Ampere's circuit law,

$$\oint \vec{H} \cdot d\vec{L} = I \qquad (1)$$

and, $$\vec{\nabla} \times \vec{H} = \vec{J} \qquad (2)$$

From Stokes Theorem, we have,

$$\oint \vec{H} \cdot \vec{dL} = \int_s (\vec{\nabla} \times \vec{H}) \cdot \vec{ds}$$

$$= \int \vec{J} \cdot \vec{ds}$$

$$\therefore \cancel{\vec{\nabla} \times \vec{H} = \vec{J}}$$

This is Maxwell's 3rd eqn with integral form.

Again,

$$\oint \vec{H} \cdot \vec{dL} = \int (\vec{\nabla} \times \vec{H}) \cdot \vec{ds}$$

$$\Rightarrow \oint (\vec{\nabla} \times \vec{H}) \cdot \vec{ds} = \int \vec{J} \cdot \vec{ds}$$

$$\therefore \vec{\nabla} \times \vec{H} = \vec{J}$$

This is Maxwell's 3rd equation with differential form or point form.

Parameter of Transmission Line

TL can be described in terms of its line parameters.

~~Rlcgc~~, which are RLGC,

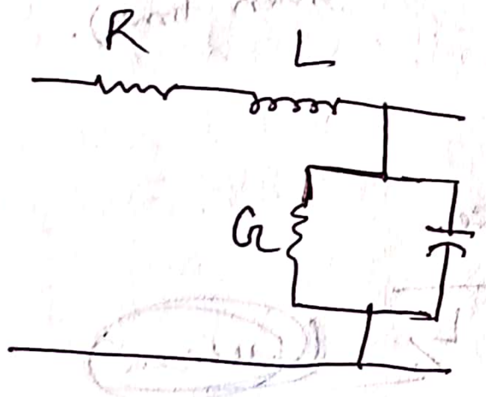
R = Resistance per unit length R

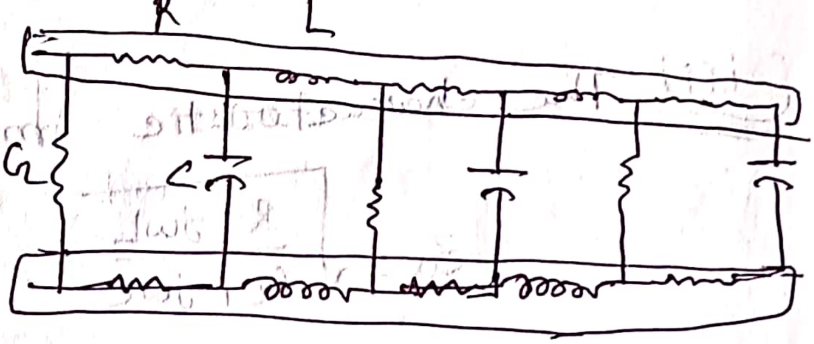
L = ~~Co~~ Inductance (आवेश), per unit length

G = Conductance " " "

C = Capacitance " " "

These all are uniformly distributed along the entire length of line.





Lossless Transmission:-

R = 0, G = 0 that means there are no ~~ohmor~~ ohmic and conduction losses in the line.

or, Conductors are perfect $\sigma_c \cong \infty$ and the dielectric material in cable is lossless.

(i) The propagation constant,

$$\gamma = \alpha + j\beta$$

$$= \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{(j\omega L)(j\omega C)}$$

$$= \sqrt{j^2\omega^2 LC}$$

$$= j\omega\sqrt{LC}$$

Trick to remember

$R+j\omega L$ = Reejul

$G+j\omega C$ → Groni Jungok

α = Attenuation constant (Reduction of signal)

β = Phase shift constant. (i.e. signals are at different ~~part~~ points of their cycle at a given time).

(iv) Velocity, $v = \frac{\omega}{\beta}$

$$= \frac{1}{\sqrt{LC}}$$

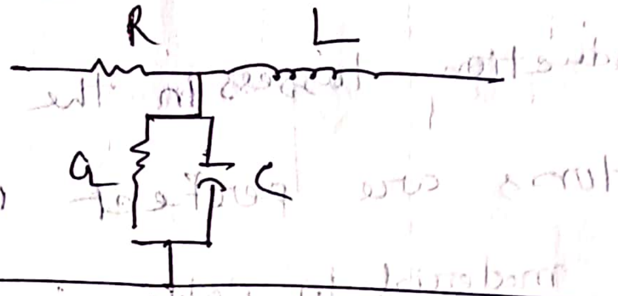
(iii) The characteristic impedance,

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{L/C}$$ (Luicee)

Distortionless Transmission Line:

Condition of distortionless:

$$R/L = G/C$$



(1) The attenuation constant (α) is independent of frequency.

(ii) The phase shift constant (β) is linearly dependent on ~~with~~ frequency.

The propagation constant,

$$\gamma = \alpha + j\beta$$

$$= \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{RG\left(1 + \frac{j\omega L}{R}\right)\left(1 + \frac{j\omega C}{G}\right)}$$

$$= \sqrt{RG\left(1 + \frac{j\omega L}{R}\right)\left(1 + \frac{j\omega L}{R}\right)}$$

$$= \sqrt{RG}\left(1 + \frac{j\omega L}{R}\right)$$

$$= \sqrt{RG} + \frac{j\omega L}{R}\sqrt{RG}$$

$$= \sqrt{RG} + j\omega L \cdot \sqrt{\frac{G}{R}}$$

$$= \sqrt{RG} + j\omega L \cdot \sqrt{C/L}$$

$$= \sqrt{RG} + j\omega\sqrt{LC}$$

$$\therefore \alpha = \sqrt{RG}$$
$$\beta = j\omega\sqrt{LC}$$

Ans

poynting Theorem & indicating various terms

Things to remember,

The modified Ampere's circuital law or Maxwell's 4th law is:-

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\delta \vec{D}}{\delta t}$$

~~From vector Calculus formula~~

$\cancel{\vec{\nabla} \cdot (\vec{E} \times \vec{H})}$

$$\Rightarrow \vec{E} \cdot (\vec{\nabla} \times \vec{H}) = \vec{E} \cdot \vec{J} + \vec{E} \cdot \frac{\delta \vec{D}}{\delta t}$$

From vector Calculus,

$$\vec{\nabla} \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot (\vec{\nabla} \times \vec{F}) - \vec{F}(\vec{\nabla} \times \vec{G})$$

$$\left.\begin{aligned} \vec{\nabla} &= \vec{E} \\ \vec{F} &= \vec{\nabla} \\ \vec{G} &= \vec{H} \end{aligned}\right\}$$

$$\vec{E}(\vec{\nabla} \times \vec{H}) = \vec{H} \cdot (\vec{E} \times \vec{\nabla}) - \vec{\nabla}(\vec{E} \times \vec{H})$$

$$\vec{H}(\vec{E} \times \vec{\nabla}) - \vec{\nabla}(\vec{E} \times \vec{H})$$
$$= \vec{E}(\vec{\nabla} \times \vec{H})$$
$$= \vec{E} \cdot \vec{J} + \vec{E} \cdot \frac{\delta \vec{D}}{\delta t}$$

P.T.O

H = Magnetic Intensity
or, (Magnetic Field strength)

E = Electric Field Strength

D = Displacement

Max এর 4th law

⇒ কোনো electric current অথবা একটি পরিবর্তনশীল electric flux একটা তলের মধ্যে যদি magnetic field তৈরি করে যেকোনো পাথ যা ঐ surface কে আবদ্ধ করে

$$\vec{\nabla} \times \vec{H} = \frac{\delta \vec{D}}{\delta t} + \vec{J}$$

$\vec{J}$ = It is the vector whose magnitude is the electric current density.

~~[illegible]~~ $\Rightarrow \nabla \times H = \vec{J} + \frac{\delta D}{\delta t}$

One of formula of vector calculus:-

$$\vec{\nabla} \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot (\vec{\nabla} \times \vec{F}) - \vec{F} \cdot (\vec{\nabla} \times \vec{G})$$

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\delta B}{\delta t}}$$

$$\Rightarrow -\nabla(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \vec{E}\cdot\frac{\delta\vec{D}}{\delta t} - \vec{H}(\vec{E}\times\vec{\nabla})$$

$$= \vec{E}\cdot\vec{J} + \vec{E}\cdot\frac{\delta\vec{D}}{\delta t} - \left(-\vec{H}\cdot\frac{\delta B}{\delta t}\right)$$

$$= \vec{E}\cdot\vec{J} + \vec{E}\cdot\frac{\delta\vec{D}}{\delta t} + \vec{H}\cdot\frac{\delta B}{\delta t}$$

$$= \vec{E}\cdot\vec{J} + \vec{E}\cdot\varepsilon\frac{\delta\vec{E}}{\delta t} + \vec{H}\cdot\mu\frac{\delta\vec{H}}{\delta t}$$

We know,
$\vec{D} = \varepsilon\vec{E}$
$\vec{B} = \mu\vec{H}$

NOW,

$$\vec{E}\cdot\frac{\delta\vec{E}}{\delta t} = \frac{\delta}{\delta t}(\vec{E}\cdot\vec{E})$$

$$\frac{d}{dx}(x\cdot y) = y\cdot\frac{d}{dx}x + x\cdot\frac{d}{dx}(y)$$

$$= \vec{E}\cdot\frac{d\vec{E}}{dt} + \vec{E}\cdot\frac{\delta\vec{E}}{\delta t}$$

$$\therefore \frac{\delta}{\delta t}(E^2) = 2\vec{E}\cdot\frac{d\vec{E}}{dt}$$

$$\therefore \vec{E}\cdot\frac{d\vec{E}}{dt} = \frac{1}{2}\cdot\frac{\delta\vec{E}^2}{\delta t}$$

Similarly, $\vec{H}\cdot\frac{\delta\vec{H}}{\delta t} = \frac{1}{2}\frac{\delta\vec{H}^2}{\delta t}$

$$\therefore -\vec{\nabla}(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \frac{1}{2}\frac{\delta\vec{E}^2}{\delta t} + \frac{1}{2}\frac{\delta\vec{H}^2}{\delta t}$$

$$\Rightarrow -\int_v \vec{\nabla}(\vec{E}\times\vec{H})\,dv = \int_v(\vec{E}\cdot\vec{J})\,dv + \int\left(\frac{1}{2}\frac{\delta\vec{E}^2}{\delta t}\right)dv + \int_v\left(\frac{1}{2}\frac{\delta\vec{H}^2}{\delta t}\right)dv$$

Applying divergence Theorem,

$$\Rightarrow -\oint_S (\vec{E} \times \vec{H}) \cdot ds = \int_v (\vec{E} \cdot \vec{J}) dv + \frac{\delta}{\delta t} \int_v \left(\frac{\varepsilon}{2} \vec{E}^2 + \frac{\mu}{2} \vec{H}^2 \right) dv$$

$$\Rightarrow \oint_S (\vec{E} \times \vec{H}) ds = -\frac{\delta}{\delta t} \int_v \left(\tfrac{1}{2} \varepsilon \vec{E}^2 + \tfrac{1}{2} \mu \vec{H}^2 \right) dv - \int_v \sigma \vec{E}^2 \, dv$$

$\oint_S (\vec{E} \times \vec{H}) ds \Rightarrow$ Total power ~~having~~ leaving the volume

$-\frac{\delta}{\delta t} \int_v \left(\tfrac{1}{2} \varepsilon \vec{E}^2 + \tfrac{1}{2} \mu \vec{H}^2 \right) dv \Rightarrow$ The rate at which energy stored in electric and magnetic fields decreases in the volume.

$- \int_v \sigma \vec{E}^2 dv \Rightarrow$ power dissipated in the volume
↳ फ़ल्हार

Lossless Dielectric

* Derive the wave equations in lossless dielectric.

Ans :-

In lossless dielectric, $\sigma = 0$, $\rho_v = 0$

Maxwell's equation in differential form,

$\vec{\nabla}\cdot\vec{D} = \rho_v$ } Gauss's law

$\vec{\nabla}\cdot\vec{B} = 0$

$\vec{\nabla}\times\vec{E} = -\frac{\delta \vec{B}}{\delta t}$

$\vec{\nabla}\times\vec{H} = J + \frac{\delta(\vec{D})}{\delta t}$

where

$\vec{D} = \varepsilon\vec{E}$

$\vec{B} = \mu\vec{H}$

$J = \sigma\vec{E}$

~~Subtit~~

Substituting,

$\vec{\nabla}\cdot\vec{D} = 0$ [∵ $\rho_v \neq 0$

$\Rightarrow \vec{\nabla}\cdot\varepsilon\vec{E} = 0$

$\Rightarrow \vec{\nabla}\cdot\vec{E} = 0$ ——— (i)

Also,

$\vec{\nabla}\cdot\vec{B} = 0$

$\Rightarrow \vec{\nabla}\cdot\mu\vec{H} = 0$

$\Rightarrow \vec{\nabla}\cdot\vec{H} = 0$ ——— (ii)

J = vector whose magnitude is electric current density

~~σ = Surphace~~

σ = Surface charge density.

ρ = volume charge densities

D = Electric Displacement

B = Magnetic Flux density

E = Electric Field vector

H = Magnetic Field intensity (ampere)

Gaussian Law,

The magnetic Flux B across any closed surface is zero

$\oint B = 0$ $\vec{\nabla}\cdot\vec{B} = 0$

(H) Magnetic Field = Region around the magnet where the moving charge experience Force.

Magnetic Flux, B = The Quantity or strength of magnetic lines produced by magnet

Again,

$$\vec{\nabla} \times \vec{E} = -\frac{\delta \vec{B}}{\delta t} \quad \text{——(3)}$$

$$\Rightarrow \cancel{\vec{\nabla}} \cancel{\vec{E}} = -\frac{\delta}{\delta t}$$

And,

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\delta \vec{D}}{\delta t} \quad \text{——(4)}$$

Taking eqⁿ (3),

$$\vec{\nabla} \times \vec{E} = -\frac{\delta \vec{B}}{\delta t}$$

using curl on both sides,

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\vec{\nabla} \times \frac{\delta \vec{B}}{\delta t}$$

$$\Rightarrow (\vec{\nabla} \cdot \vec{E}) \cdot \vec{\nabla} - (\vec{\nabla} \cdot \vec{\nabla}) \cdot \vec{E} = \vec{\nabla} \times \frac{\delta \vec{B}}{\delta t} \qquad [A \times (B \times C) = (A \cdot C) B - (A \cdot B) \cdot C]$$

$$\Rightarrow 0 - \vec{\nabla}^2 \vec{E} = \vec{\nabla} \times \frac{\delta \mu \vec{H}}{\delta t} \qquad [\because \vec{\nabla} \cdot \vec{E} = 0 \text{ সমীকরণ (1)}]$$

$$\Rightarrow -\vec{\nabla}^2 \vec{E} = \mu \frac{\delta}{\delta t} (\vec{\nabla} \times \vec{H})$$

$$\Rightarrow -\vec{\nabla}^2 \vec{E} = \mu \frac{\delta}{\delta t} \left(\frac{\delta \vec{D}}{\delta t} \right)$$

$$\Rightarrow \boxed{\vec{\nabla}^2 \vec{E} = \mu \frac{\delta^2 \mu \varepsilon \vec{E}}{\delta t^2}}$$

And this is call the equation in terms of $\vec{E}$.

Sinosoidal time variation. on time harmonic Form,

$$\frac{\delta}{\delta t} = j\omega$$

$$\therefore \cancel{\nabla} - \vec{\nabla}^2 . \vec{E} = \mu \frac{\delta}{\delta t} \cdot \frac{\delta \varepsilon \vec{E}}{\delta t}$$

$$= \mu (j\omega)(j\omega) \cdot \varepsilon \vec{E}$$

$$= -\mu \omega^2 \varepsilon \vec{E} \qquad [\because j^2 = -1]$$

Now, taking eq^n-(4)

$$\vec{\nabla} \times \vec{H} = J + \frac{\delta \vec{D}}{\delta t}$$

Taking curl on both side,

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{H}) = \cancel{J+} \vec{\nabla} \times \left(\cancel{J+} \frac{\delta \vec{D}}{\delta t} \right)$$

$$\Rightarrow (\vec{\nabla} \cdot \vec{H}) . \vec{\nabla} - (\vec{\nabla} \cdot \vec{\nabla}) \cdot \vec{H} = \frac{\delta}{\delta t} (\vec{\nabla} \times \vec{D})$$

$$\Rightarrow 0 - \vec{\nabla}^2 \vec{H} = \frac{\delta}{\delta t} (\vec{\nabla} \times \varepsilon \vec{E})$$

$$\Rightarrow -\vec{\nabla}^2 \vec{H} = \varepsilon \frac{\delta}{\delta t} (\vec{\nabla} \times \vec{E})$$

$$\Rightarrow -\vec{\nabla}^2 \vec{H} = \varepsilon \frac{\delta}{\delta t} \left(-\frac{\delta \vec{B}}{\delta t} \right)$$

$$\Rightarrow -\vec{\nabla}^2 . \vec{H} = -\varepsilon \frac{\delta^2}{\delta t^2} (\mu \vec{H})$$

$$\Rightarrow \vec{\nabla}^2 . \vec{H} = \mu \varepsilon \frac{\delta^2}{\delta t} (\vec{H})$$

This eq^n is called wave eq^n in terms of $\vec{H}$

In sinosoidal form,

we know,

$$\frac{\delta}{\delta t} = j\omega$$

$$\Rightarrow \vec{\nabla}^2 \cdot \vec{H} = \mu\varepsilon (j\omega)(j\omega)\vec{H}$$

$$= -\omega^2 \mu\varepsilon \vec{H}$$

$$\boxed{\text{wave } eq^n \text{ for conducting medium} \quad \text{Lossy medium}}$$

The conducting medium $\sigma \neq 0$, $\rho_v = 0$

Maxwell's eq^n in diff form,

$\vec{\nabla} \cdot \vec{D} = \rho_v$ — ①

$\vec{\nabla} \cdot \vec{B} = 0$ — ②

$\vec{\nabla} \times \vec{E} = -\frac{\delta \vec{B}}{\delta t}$ — ③

$\vec{\nabla} \times \vec{H} = \vec{j} + \frac{\delta \vec{D}}{\delta t}$ — ④

where

$\vec{D} = \varepsilon \vec{E}$

$\vec{B} = \mu \vec{H}$

$\vec{j} = \sigma \vec{E}$

Substituting eq^n,

$\vec{\nabla} \cdot \vec{D} = \rho_v$

$\Rightarrow \vec{\nabla} \cdot \vec{D} = 0 \Rightarrow \vec{\nabla} \cdot \varepsilon \vec{E} = 0 \Rightarrow \vec{\nabla} \cdot \vec{E} = 0$ — ①

Then,

$$\vec{\nabla}.\vec{B} = 0$$

$$\Rightarrow \vec{\nabla}.\ \mu\vec{H} = 0 \qquad \Rightarrow \vec{\nabla}.\vec{H} = 0 \quad \text{—— (2)}$$

Again,

$$\vec{\nabla}\times\vec{E} = -\frac{\delta\vec{B}}{\delta t} \quad \text{—— (3)}$$

Also,

$$\vec{\nabla}\times H = J + \frac{\delta\vec{D}}{\delta t} \quad \text{—— (4)}$$

Taking eqn (3),

$$\vec{\nabla}\times\vec{E} = -\frac{\delta.\vec{B}}{\delta t}$$

Adding curl on both side,

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\times\left(-\frac{\delta\vec{B}}{\delta t}\right)$$

$$\Rightarrow (\vec{\nabla}\times.\vec{E})\cdot\vec{\nabla} - (\vec{\nabla}.\vec{\nabla}).\vec{E} = -\frac{\delta}{\delta t}(\vec{\nabla}\times\vec{B})$$

$$\Rightarrow 0 - \vec{\nabla}^2\vec{E} = -\frac{\delta}{\delta t}(\vec{\nabla}\times\mu\vec{H})$$

$$\Rightarrow -\vec{\nabla}^2.\vec{E} = -\mu\frac{\delta}{\delta t}(\vec{\nabla}\times\vec{H})$$

$$\Rightarrow -\vec{\nabla}^2.\vec{E} = -\mu\frac{\delta}{\delta t}\left(J + \frac{\delta\vec{D}}{\delta t}\right)$$

$$\Rightarrow \vec{\nabla}^2.\vec{E} = \mu\frac{\delta}{\delta t}\left(\sigma\vec{E} + \frac{\delta\vec{D}}{\delta t}\right)$$

$$\Rightarrow \vec{\nabla}^2 \cdot \vec{E} = \mu \frac{\delta}{\delta t}\left(\sigma \vec{E} + \frac{\delta \varepsilon \vec{E}}{\delta t}\right)$$

$$= \mu \sigma \frac{\delta}{\delta t} \vec{E} + \mu \varepsilon \frac{\delta^2 \vec{E}}{\delta t^2}$$

(Lossy medium) In terms of $\vec{E}$

Conducting Equation in terms of $\vec{E}$ for sinusoidal form :-

we know

$$\frac{\delta}{\delta t} = j\omega$$

so,

$$\vec{\nabla}^2 \vec{E} = \mu \sigma (j\omega) \vec{E} = \mu \varepsilon j^2 \omega^2 \vec{E}$$

$$\Rightarrow \vec{\nabla}^2 \vec{E} = j\omega\mu \left(\sigma + (\varepsilon j\omega)\right) \vec{E}$$

$$\therefore \vec{\nabla}^2 \vec{E} = \gamma^2 \vec{E} \qquad \gamma = \alpha + j\beta$$

$$\therefore \gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} \Rightarrow \text{propagation delay}$$

propagation constant

This is the wave equation ~~& is called propagation Constant of~~ in terms of $\vec{E}$ for sinusoidal ~~of the medium~~ time variation for conducting medium.

Now taking eqⁿ-(4)

$$\vec{\nabla}\times\vec{H} = J + \frac{\delta \vec{D}}{\delta t}$$

Taking curl on both sides,

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\times\left(J + \frac{\delta \vec{D}}{\delta t}\right)$$

$$\Rightarrow (\vec{\nabla}\cdot\vec{H})\vec{\nabla} - (\vec{\nabla}\cdot\vec{\nabla})\vec{H} = \vec{\nabla}\times J + \frac{\delta}{\delta t}(\vec{\nabla}\times \vec{D})\ \varepsilon\vec{E})$$

$$\Rightarrow 0 - \vec{\nabla}^2\vec{H} = \vec{\nabla}\times(\sigma\cdot\vec{E}) + \varepsilon\frac{\delta}{\delta t}\left(\cancel{\frac{\delta \vec{E}}{\delta t}}\right)(\vec{\nabla}\times\vec{E})$$

$$\Rightarrow -\vec{\nabla}^2\vec{H} = \sigma(\vec{\nabla}\times\vec{E}) + \cancel{\frac{\delta}{\delta t}\left(\frac{\delta \vec{E}}{\delta t}\right)}\ \varepsilon\frac{\delta}{\delta t}\left(-\frac{\delta \vec{B}}{\delta t}\right)$$

$$\Rightarrow -\vec{\nabla}^2\cdot\vec{H} = \sigma\left(-\frac{\delta \vec{B}}{\delta t}\right) + \varepsilon\frac{\delta}{\delta t}\left(-\frac{\delta \vec{B}}{\delta t}\right)$$

$$\Rightarrow -\vec{\nabla}^2\cdot\vec{H} = -\sigma\left(\frac{\delta \mu\vec{H}}{\delta t}\right) - \varepsilon\frac{\delta^2}{\delta t^2}(\mu\vec{H})$$

$$\Rightarrow \vec{\nabla}^2\vec{H} = \mu\sigma\left(\frac{\delta \vec{H}}{\delta t}\right) + \mu\varepsilon\frac{\delta^2}{\delta t^2}(\vec{H})$$

This is the eqⁿ in term of $\vec{H}$.

In terms of $\vec{H}$ sinusoidal form:-

$$\frac{\delta}{\delta t} = j\omega$$

$$\Rightarrow \vec{\nabla}^2\vec{H} = \mu\sigma(j\omega)\vec{H} + \mu\varepsilon(j\omega)^2(\vec{H})$$

$$= j\omega\mu(\sigma + \varepsilon j\omega)\vec{H}$$

$$\therefore \vec{\nabla}^2\vec{H} = P^2\vec{H}$$

Attenuation & phase Shift Constant

(*) Derive the expression for the attenuation & phase shift constants in a lossy dielectric medium.

$\left(j^2 = -1 \right)$

α = Attenuation Constant

β = phase shift constant.

⇒ For lossy dielectric, $\sigma \neq 0$

The propagation constant,

$$\gamma = \alpha + j\beta$$

$$= \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

$$\text{or, } (\alpha + j\beta)^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

$$\text{or, } \alpha^2 + 2j\beta.\alpha - \beta^2 = j\omega\mu\sigma - \omega^2\mu\varepsilon$$

$$\cancel{\text{or, } \alpha^2 - \beta^2 + \omega^2\mu\varepsilon = j\omega\mu\sigma - 2j\beta\alpha}$$

Real part,

$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon \quad \text{—(i)}$$

Imaginary part,

$$2j\beta.\alpha = j\omega\mu.\sigma$$

$$\text{or, } \beta = \frac{j\omega\mu\sigma}{2j\alpha}$$

$$= \frac{\omega\mu\sigma}{2\alpha} \quad \text{—(ii)}$$

⇒ সমীকরণ (ii) এর value (i) এ বসাই,

$$\alpha^2 - \left(\frac{\omega\mu\sigma}{2\alpha}\right)^2 = -\omega^2\mu\varepsilon$$

or, $\alpha^4 - \left(\frac{\omega\mu\sigma}{2}\right)^2 = -\omega^2\mu\epsilon\alpha^2$

or, $(\alpha^2)^2 + \omega^2\mu\epsilon\alpha^2 = \left(\frac{\omega\mu\sigma}{2}\right)^2$

or, $(\alpha^2)^2 + 2\cdot\alpha^2\cdot\frac{1}{2}\,\omega^2\mu\epsilon + \left(\frac{\omega^2\mu\epsilon}{2}\right)^2 - \left(\frac{\omega^2\mu\epsilon}{2}\right)^2 = \left(\frac{\omega\mu\sigma}{2}\right)^2$

or, $\left(\alpha^2 + \frac{\omega^2\mu\epsilon}{2}\right)^2 = \frac{\omega^2\mu^2\sigma^2}{4} + \frac{\omega^4\mu^2\epsilon^2}{4}$

or, $\left(\alpha^2 + \frac{\omega^2\mu\epsilon}{2}\right)^2 = \frac{\omega^4\mu^2\epsilon^2}{4}\left(1 + \frac{\cancel{\omega}\,\sigma^2}{\omega^2\epsilon^2}\right)$

or, $\left(\alpha^2 + \frac{\omega^2\mu\epsilon}{2}\right) = \frac{\omega^2\mu\epsilon}{2}\sqrt{\left(1 + \frac{\sigma^2}{\omega^2\epsilon^2}\right)}$

or, $\alpha^2 = \frac{\omega^2\mu\epsilon}{2}\sqrt{\left(1 + \frac{\sigma^2}{\omega^2\epsilon^2}\right)} - \frac{\omega^2\mu\epsilon}{2}$

or, α =

~~∴ Attenuation constant~~ $\cancel{= \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1 + \frac{\sigma^2}{\omega^2\epsilon^2}}\right) - \frac{\omega^2\mu\epsilon}{2}}}$

or, $\alpha^2 = \frac{\omega^2\mu\epsilon}{2}\left(\sqrt{1 + \frac{\sigma^2}{\omega^2\epsilon^2}} - 1\right)$

Attenuation constant, $\alpha = \omega\sqrt{\cancel{\omega}\,\frac{\mu\epsilon}{2}\left(\sqrt{1 + \frac{\sigma^2}{\omega^2\epsilon^2}} - 1\right)}$

Now,

$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon$$

$$\Rightarrow \beta^2 = \alpha^2 + \omega^2\mu\varepsilon$$

$$= \frac{\omega^2\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} - 1\right) + \omega^2\mu\varepsilon$$

$$= \frac{\omega^2\mu\varepsilon}{2}\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} - \frac{\omega^2\mu\varepsilon}{2} + \omega^2\mu\varepsilon$$

$$= \frac{\omega^2\mu\varepsilon}{2}\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} + \frac{\omega^2\mu\varepsilon}{2}$$

$$= \frac{\omega^2\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} + 1\right)$$

$\therefore$ Phase shift constant, $\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} + 1\right)}$

Conductors & Insulators

Conduction Current Density, $\vec{J}_c = \sigma \vec{E}$

Displacement " " , $\vec{J}_D = \frac{\partial \vec{D}}{\partial t} = j\omega \vec{D} = j\omega\epsilon \vec{E}$

$$\left|\frac{\vec{J}_c}{\vec{J}_D}\right| = \left|\frac{\sigma \vec{E}}{j\omega\epsilon \vec{E}}\right| = \frac{\sigma}{\omega\epsilon}$$

$\frac{\sigma}{\omega\epsilon} >> 1$ or, $\sigma >> \omega\epsilon$ that is good conduction.

$\frac{\sigma}{\omega\epsilon} << 1$ or, $\sigma << \omega\epsilon$ good insulator.

$\boxed{\text{Derivation of } \alpha, \beta, u \text{ \& } \eta}$:-

For good dielectric, $\sigma/\omega\epsilon << 1$ or, $\sigma << \omega\epsilon$

$$\boxed{\text{The attenuation constant, } \alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}} - 1\right)}}$$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots \rightarrow$ [Binomial Expression]

$\left(1+\frac{\sigma^2}{\omega^2\epsilon^2}\right)^{1/2} = 1 + \frac{1}{2}\frac{\sigma^2}{\omega^2\epsilon^2}$ [Neglecting Higher Order Terms]

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(1+\frac{\sigma^2}{2\omega^2\epsilon^2} - 1\right)} = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\frac{\sigma^2}{2\omega^2\epsilon^2}\right)}$$

$$= \omega\frac{\sqrt{\mu}\sqrt{\epsilon}\,\sigma}{2\omega\epsilon} = \frac{\sqrt{\mu}\,\sigma}{2\sqrt{\epsilon}} \quad \therefore \alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$$

The phaseshift constant,

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right)}$$

$$(1+x)^n = 1+nx + \frac{n(n-1)}{2!}\cdot x^2 + \cdots$$

$$\left(1+\frac{\sigma^2}{\omega^2\varepsilon^2}\right)^{1/2} = 1+\frac{1}{2}\frac{\sigma^2}{\omega^2\varepsilon^2}+\cdots$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(1+\frac{1}{2}\frac{\sigma^2}{\omega^2\varepsilon^2}+1\right)}$$

$$= \omega\sqrt{\frac{\mu\varepsilon}{2}\left(2+\frac{\sigma^2}{2\omega^2\varepsilon^2}\right)}$$

$$= \omega\sqrt{\frac{\omega\varepsilon}{2}\times 2\left(1+\frac{\sigma^2}{4\omega^2\varepsilon^2}\right)}$$

$$= \omega\sqrt{\omega\varepsilon\left(1+\frac{\sigma^2}{4\omega^2\varepsilon^2}\right)}$$

$$= \omega\sqrt{\omega\varepsilon}\left(1+\frac{\sigma^2}{4\omega^2\varepsilon^2}\right)^{\cancel{2}\cdot 1/2}$$

$$\therefore \beta = \omega\sqrt{\omega\varepsilon}\left(1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$$

Intrinsic Impedance, $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$

$$= \sqrt{\frac{j\omega\mu}{j\omega\varepsilon\left(1 + \frac{\sigma}{j\omega\varepsilon}\right)}}$$

$$= \sqrt{\frac{\mu}{\varepsilon}}\left(1 + \frac{\sigma}{j\omega\varepsilon}\right)^{\frac{1}{2}}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} \cdot \left(1 + \left(\tfrac{1}{2}\right)\frac{\sigma}{j\omega\varepsilon}\right) = \sqrt{\frac{\mu}{\varepsilon}}\left(1 - \frac{\sigma}{j \cdot 2\omega\varepsilon}\right)$$

$$= \sqrt{\frac{\mu}{\varepsilon}}\left(1 + \frac{\sigma j}{2\omega\varepsilon}\right)$$

If $\varepsilon_r = 81$, $\mu = \mu_0$ for the medium in which a wave with a frequency of $0.3\,GHz$ is propagating. Determine the propagation constant & intrinsic impedance of the medium when $\sigma = 0$.

$\Rightarrow$ $\gamma = \alpha + j\beta$

$$= \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

(jammu kashmir (+)
Jammu sigma kintu
jammu silent
$j\omega$ ε)

$$= \sqrt{j\omega\mu(j\omega\varepsilon)}$$

$$= \sqrt{j^2\omega^2\mu\varepsilon} = j\omega\sqrt{\mu\varepsilon}$$

$$= j\,2\pi f\sqrt{\mu_0\varepsilon_0\varepsilon_r}\left[\because \omega = 2\pi f\right]$$

We know

$\sqrt{\mu_0\varepsilon_0} = \frac{1}{3\times10^8}$

$\sqrt{\frac{\mu_0}{\varepsilon_0}} = 377$

$$= j\,2\pi\,(0.3\times10^{9}) \times \sqrt{\tfrac{1}{3\times10^{8}}\times 9} \quad \tfrac{1}{3\times10^{8}}\sqrt{9}$$

$$= j\,2\pi\,(0.3\times10^{9}) \times \frac{1}{3\times10^{8}} \times 3$$

$$= j\,6\pi = j\times 6\times 3.1416$$

$$= j\,18.85\ m^{-1}$$

Now,

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r}}$$

$$= \sqrt{\frac{\mu_0}{\varepsilon_0}}\cdot\frac{1}{\sqrt{\varepsilon_r}} = \frac{377}{3}$$

$$= 125.127\ \Omega$$

$\boxed{\text{Part - A}}$

Conversion of Differential Form Of Maxwell's Equation to Integral Form

Diff Form		Integral Form
(1) $\vec{\nabla} \times \vec{H} = \dot{\vec{D}} + \vec{J}$	$\longrightarrow$	$\oint_L \vec{H} \cdot d\vec{L} = \int_s (\dot{\vec{D}} + \vec{J})\, d\vec{s}$
(2) $\vec{\nabla} \times \vec{E} = -\dot{\vec{B}}$	$\longrightarrow$	$\oint \vec{E} \cdot d\vec{L} = -\int_s \dot{\vec{B}}\, d\vec{s}$
(3) $\vec{\nabla} \cdot \vec{D} = \rho_v$	$\longrightarrow$	$\oint \vec{D} \cdot ds = \int_v \rho_v\, dv$
(4) $\vec{\nabla} \cdot \vec{B} = 0$	$\longrightarrow$	$\oint_s \vec{B} \cdot d\vec{s} = 0$

$\boxed{\text{PROOF}}$

(1) 1st Equation:-

$$\vec{\nabla} \times \vec{H} = \dot{\vec{D}} + \vec{J}$$

$$\Rightarrow \int_s (\vec{\nabla} \times \vec{H}) ds = \int_s (\dot{\vec{D}} + \vec{J})\, d\vec{s}$$

$$\Rightarrow \int_s (\vec{\nabla} \times \vec{H}) ds = \oint_L \vec{H} \cdot d\vec{L}$$

$$\Rightarrow \oint_L \vec{H} \cdot d\vec{L} = \int_s (\dot{\vec{D}} + \vec{J})\, d\vec{s}$$

② $\vec{\nabla} \times \vec{E} = -\vec{B}$

$\Rightarrow \int_S \vec{\nabla} \times \vec{E}\, d\vec{s} = -\oint_S \vec{B} \cdot d\vec{s}$

$\Rightarrow \int_S (\vec{\nabla} \times \vec{B})\, ds = \oint_L \vec{E} \cdot d\vec{L}$

$\Rightarrow \oint_L \vec{E} \cdot dL = -\int_S \vec{B} \cdot d\vec{s}$

③ $\vec{\nabla} \cdot \vec{D} = \rho_v$

$\Rightarrow \int_V \vec{\nabla} \cdot \vec{D}\, dv = \int_V \rho_v\, dv$

$\Rightarrow \int_V (\vec{\nabla} \cdot \vec{D})\, dv = \oint_S \vec{D} \cdot d\vec{s}$

$\Rightarrow \oint_S \vec{D} \cdot d\vec{s} = \int_v \rho_v\, dv$

④ $\vec{\nabla} \cdot \vec{B} = 0$

$\int_v \vec{\nabla} \cdot \vec{B}\, dv = 0$

$\int_v (\vec{\nabla} \cdot \vec{B})\, dv = \oint_S \vec{B} \cdot d\vec{s}$

$\therefore \oint_S \vec{B} \cdot d\vec{s} = 0$